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$\bar{\mu}$ - MESIC MOLECULAR IONS AND $\bar{\mu}$ - MESIC MOLECULAR PROCESSES IN HYDROGEN

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μ MESIC MOLECULAR IONS AND μ - MESIC MOLECULAR PROCESSES IN HYDROGEN

ABSTRACT

A number of atomic and mesic molecular processes in the medium of hydrogen isotopes are considered (molecular ion formation $H_\mu + H^{(i)} \to (H^{(i)} H^{(k)})_\mu$ elastic collisions and charge exchange of mesic atoms:

$$H^{(1)}_\mu + H^{(2)+} \to H^{(1)}_\mu + H^{(2)+}, \quad H^{(1)}_\mu + H^{(2)+} \to H^{(1)+} + H^{(2)}_\mu$$

Energy levels of mesic molecules are calculated. The numerical calculations were carried out on BESM computer. Corrections due to the motion of nuclei up to the first order with respect to $\frac{m_\mu}{M}$ are taken into account.

A special property of mesic hydrogen atoms is their neutrality since at distances larger compared to the radius of Bohr mesic atom orbit ($2.56 \cdot 10^{-11}$ cm) the nucleus charge is almost completely screened by the meson charge. This circumstance provides a number of mesic molecular processes in hydrogen (or in the mixture of hydrogen isotopes) such as μ -meson exchange between various nuclei (charge exchange), formation of mesic molecules etc. These processes define to a large extent the catalysis of nuclear reactions in hydrogen predicted by Frank[1], Zeldovich[2] and Sacharov[3] and investigated experimentally in papers[4,5]. On the other hand as it was pointed out in[6-8], mesic molecular processes in hydrogen are important in the experimental determination of the law of interaction $\mu + p \to n + \nu$ (in particular, in order to distinguish experimentally $V - A$ and $V + A$ forms[9]).

Some mesic molecular processes in hydrogen have been studied in earlier papers.

In the paper[11] the levels of mesic molecular ions $(pp)^+_\mu$; $(pd)^+_\mu$; $(dd)^+_\mu$; and cross sections of the processes

$$p_\mu + p \to (pp)^+_\mu, \quad d_\mu + p \to (pd)^+_\mu, \quad d_\mu + p \to d_\mu + p$$

have been calculated. There is a great discrepancy with our data for the upper level of $(dd)_\mu$ with $L = 0$. Our cross section of the scattering of d_μ by protons does not tend to zero with energy decrease.

In [12] the probability of the charge exchange $p_\mu + d \to d_\mu + p$ has been calculated by the method similar to that used in the present paper. However for the potentials f_g and f_u and for the corrections K_{gg} and K_{uu} more rough functions than those given in [14,15] has been taken. Moreover, the authors[12] have expected that for $R \geq 6$ the exact solutions of the system coincide with their asymptotic values, what is not quite correct.

In paper [18] the ground state levels in mesic molecular ions have been calculated. The corrections to the potential energy of order $\frac{m_\mu}{M}$ have been neglected. Besides, this in order to find the eigenvalues for mesic molecules with different nuclei we need to solve a system of two equations (owing to the presence of a dipole moment which produces transitions between the states g and u).

In [20] the levels in mesic molecular ions $(pt)_\mu$ $(td)_\mu$ and $(tt)_\mu$ are determined (the

corrections due to the motion of the [illegible] equation for the mesic function is less accurate than in [14].

In paper [14] the estimates of basic levels for mesic molecular ions and reactions $d + p_\mu \to d_\mu + p,\ d_\mu + p + (pd)_\mu$ are given.

Let us consider a system consisting of two nuclei of hydrogen isotopes with masses M_1 and M_2 and the μ^--meson. Let $\vec{r}$, $\vec{R}_1$, $\vec{R}_2$ be the coordinates of μ^- meson and nuclei*. The Hamiltonian of the system (if the spin interactions are neglected) is of the form:

$$\hat{H} = -\frac{1}{2}\Delta_\mu - \frac{1}{2M_1}\Delta_{R_1} - \frac{1}{2M_2}\Delta_{R_2} - \frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{R} \qquad (1)$$

where

$$r_1 = |\vec{r} - \vec{R}_1|; \quad r_2 = |\vec{r} - \vec{R}_2|, \quad R = |\vec{R}_2 - \vec{R}_1|$$

Believing that the μ^--meson is on the K-orbit we shall seek for the wave function of the system in the form:

$$\Psi = \mathcal{Y}(\vec{R})\,\Sigma_g(R,\vec{r}) + H(\vec{R})\,\Sigma_n(R,\vec{r}) \qquad (2)$$

where $\mathcal{Y}(\vec{R})$ and $H(\vec{R})$ describe the relative motion of nuclei, and Σ_g and Σ_n represent the wave functions of μ^- meson in the field of fixed nuclei which are from one other at the distance R **. When $R \to \infty$

$$\Sigma_g \to \frac{1}{\sqrt{2\pi}}\left(e^{-r_1} + e^{-r_2}\right);\ \Sigma_n \to \frac{1}{\sqrt{2\pi}}\left(e^{-r_1} - e^{-r_2}\right) \qquad (3)$$

and when $R \to 0$ Σ_g and Σ_n transform into the wave functions of the H_e ion states $1S$ and $2P$ correspondingly.

Substituting (2) into the Schrödinger equation

$$\hat{H}\Psi = E\Psi \qquad (4)$$

and having in mind that the wave functions Σ_g and Σ_n satisfy the equation:

$$\left(-\frac{1}{2}\Delta_r - \frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{R}\right)\Sigma_i = E_i(R)\,\Sigma_i(\vec{r}, R) \qquad (5)$$

we shall obtain after multiplying by Σ_g and Σ_n and integrating over the μ^--meson coordinates the system of equations for $\mathcal{Y}(\vec{R})$ and $H(\vec{R})$.

---

* Further we shall use the mesic atom units $\hbar = 1$, $e = 1$, $m_\mu = 207 m_e = 1$. The mesic atom unit of length is $\hbar^2/m_\mu e^2 = 2.56\ 10^{-11}$ cm while the mesic atom unit of energy $m_\mu e^4/\hbar^2 = 5.6 \cdot 10^3$ ev).

** Let us note that since the wave function (2) depends only on the differences of particle coordinates in the center-of-mass system the chosen coordinate system is at rest.

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$$-\frac{1}{2M_{12}}\Delta_{\vec{R}}\,\varphi+\left(E_g+\frac{1}{2M_{12}}K_{gg}\right)\varphi+\frac{1}{2M_{12}}K_{gu}H-\frac{1}{M_{12}}Q_{gu}\nabla_R H=E\varphi$$

$$-\frac{1}{2M_{12}}\Delta_{\vec{R}}H+\left(E_u+\frac{1}{2M_{12}}K_{uu}\right)H+\frac{1}{2M_{12}}K_{ug}\varphi-\frac{1}{M_{12}}Q_{ug}\nabla_R\varphi=EH \qquad (6)$$

where $1/M_{12}=1/M_1+1/M_2$; $\vec{R}=\vec{R}_2-\vec{R}_1$ and the functions $\frac{1}{2M_{12}}K_{ig}$; $\frac{1}{M_{12}}Q_{ig}$ are the matrix elements between the functions Σ_g and Σ_u of operators

$$\frac{1}{2M_{12}}\hat{K}=-\frac{1}{2}\left(\frac{1}{M_1}\Delta_{\vec{R}_1}+\frac{1}{M_2}\Delta_{\vec{R}_2}\right) \qquad (7)$$

$$\frac{1}{M_{12}}\hat{Q}=\left(-\frac{1}{M}\nabla_{\vec{R}_1}+\frac{1}{M_2}\nabla_{\vec{R}_2}\right) \qquad (8)$$

It is easy to show that due to the normalization conditions of Σ_g and Σ_u the matrix elements of the operator $\hat{Q}$ are equal zero while the non-diagonal ones are opposite in sign in view of the orthogonality of Σ_g and Σ_u

$$Q_{gu}=-Q_{ug}=Q \qquad (9)$$

If we use the property of symmetry of Σ_g and Σ_u with respect to the exchange of nuclei we can separate the dependence on masses in matrix elements

$$K_{ii}=\int\Sigma_i(-\Delta_{\vec{R}_1})\Sigma_i\,d\tau \qquad i=g,u \qquad (10)$$

$$K_{ij}=\frac{M_2-M_1}{M_2+M_1}\int\Sigma_i(-\Delta_{\vec{R}_1})\Sigma_j\,d\tau \qquad (11)$$

$$\vec{Q}=\frac{M_2-M_1}{M_2-M_1}\int\Sigma_g(-\vec{\nabla}_{\vec{R}})\Sigma_u\,d\tau=Q\frac{\vec{R}}{R} \qquad (12)$$

Thus, if the nuclei are identical $M_1=M_2$ the system of equations (6) breaks up into two independent equations. This result is quite clear since the wave function (2) for identical nuclei in virtue of the symmetry can include only one term (either Σ_g or Σ_u). The terms $\frac{1}{2M_{12}}K_{ij}(R)$, $\frac{1}{M_{12}}Q(R)$ represent themselves the corrections to the adiabatic potentials due to the motion of nuclei (with the accuracy up to the first order with respect to m/M_{12}). Since the ratio m/M_{12} for the meson is not so small as for the electron the above terms contribute

essentially. For $R \to \infty$ the value $Q \to 0$ and the terms $\frac{1}{2M_{12}} K_{ij}(\infty)$ represent the corrections which take into account the reduced masses of the separated mesic atoms with the accuracy $\left(\frac{m_\mu}{M_1}\right)^2, \left(\frac{m_\mu}{M_2}\right)^2$. Noting that the energy of separated mesic atoms (in mesic atom units) is

$$
\begin{aligned}
E_1^0 &= -\frac{1}{2}\frac{M_1}{M_1+1} \approx -\frac{1}{2} + \frac{1}{2M_1}, \\
E_2^0 &= -\frac{1}{2}\frac{M_2}{M_2+1} \approx -\frac{1}{2} + \frac{1}{2M_2},
\end{aligned}
\tag{13}
$$

and taking into account that the matrix elements of the operator $(-\Delta_{\vec{R}})$ for $R \to \infty$ equal ½ (Appendix) we can write:

$$\left\{E_g(\infty) + \frac{1}{2M_{12}} K_{gg}(\infty)\right\} \left\{E_u(\infty) + \frac{1}{2M_{12}} K_{uu}(\infty)\right\} \frac{1}{2}(E_1^0 + E_2^0) \tag{14}$$

$$\frac{1}{2M_{12}} K_{gu}(\infty) = \frac{1}{2M_{12}} K_{gu}(\infty) \; \frac{1}{2}(E_1^0 - E_2^0) = \frac{1}{2}\Delta E \tag{15}$$

Separating in the quantities $E_g(R), E_u(R)$ and $\frac{1}{2M_{12}} K_{ij}(R)$ their values for $R = \infty$

$$
\begin{aligned}
E_{g,u}(R) &= E_{g,u}(\infty) + E'_{g,n}(R) \\
K_{ij}(R) &= K_{ij}(\infty) + K_{ij}(R)
\end{aligned}
\tag{16}
$$

we rewrite the system of equations (6) in the form

$$
\begin{aligned}
&-\frac{1}{2M_{12}}\Delta_R \varphi + \left(E' + \frac{1}{2M_{12}} K_{gg}'\right)\varphi + \left(\frac{1}{2}\Delta E + \frac{1}{2M_{12}} K_{gu}\right) H - \frac{1}{M_{12}} Q \frac{dH}{dR} = \\
&-\frac{1}{2M_{12}}\Delta_R H + \left(E_u' + \frac{1}{2M_{12}} K_{uu}\right) H + \left(\frac{1}{2}\Delta E + \frac{1}{2M_{12}} K'_{ug}\right)\varphi + \frac{1}{M_{12}} Q \frac{d\varphi}{dR} = E'H
\end{aligned}
\tag{6a}
$$

where $\Delta E = E_1^0 - E_2^0$ and the energy E' is calculated from the middle between the levels of separated mesic atoms:

$$E' = E - \frac{1}{2}(E_1^0 + E_2^0) \tag{17}$$

(When the nuclei are identical E' is calculated simply from the level of separate mesic atom).

Separating the angular dependence $\varphi(\vec{R})$ and $H(\vec{R})$

$$g_L(\vec{R}) = \frac{1}{R} g_L(R) Y_{L,M_L}(\theta,\varphi)$$
$$h_L(\vec{R}) = \frac{1}{R} h_L(R) Y_{L,M_L}(\theta,\varphi)$$
(18)

(where $Y_{L,M_L}(\theta,\varphi)$ is the spherical function) we obtain for $g(R)$ and for $h(R)$ the equations:

$$-\frac{1}{2M_{12}}\frac{d^2 g_L}{dR^2} + \left(E_g' + \frac{1}{2M_{12}}K_{gg}' + \frac{L(L+1)}{2M_{12}R^2}\right)g_L + \left(\frac{1}{2}\Delta E + \frac{1}{2M_{12}}K_{gu}'\right)h_L - \frac{1}{M_{12}}Q\cdot R\frac{d}{dR}\left(\frac{h}{R}\right) = Eg_L,$$

$$-\frac{1}{2M_{12}}\frac{d^2 h_L}{dR^2} + \left(E_u' + \frac{1}{2M}K_{uu}' + \frac{L(L+1)}{2M_{12}R^2}\right)h_L + \left(\frac{1}{2}\Delta E + \frac{1}{2M_{12}}K_{ug}'\right)g + \frac{1}{M_{12}}QR\frac{d}{dR}\left(\frac{g_L}{R}\right) = Eh_L$$
(19)

The potentials $E_g(R)$ and $E_u(R)$ were determined by solving the Eqs. (5) by many authors, starting with the paper [17]. In the present paper the values $E_g(R)$ and $E_u(R)$ taken from [14] have been used. The values of the functions $K_{gg}'(R)$ and $K_{uu}'(R)$ may be got by means of the recalculation from [15] and the values $Q(R), K_{ug}'(R), K_{gu}'(R)$ are calculated in the approximation of 'united atom' (UA) and 'linear combination of atomic orbits' (LCAO) respectively for small and large values of R (for details see Appendix). For investigating the asymptotic behaviour of the solution for $R \to \infty$ it is convenient to introduce the functions

$$a_L(R) = \{(g_L + h_L)\}/\sqrt{2}; \quad b_L(R) = \{g_L(R) - h_L(R)\}/\sqrt{2} \qquad (20)$$

Comparing (2) and (3) it is easy to see that the functions $a(R)$ and $b(R)$ for $R \to \infty$ describe a radial motion of the nucleus with mass M_2 with respect to mesic atom M_1 and of the nucleus with mass M_1 with respect to mesic atom with mass respectively M_2.

The functions (20) satisfy the equations:

$$-\frac{1}{2M_{12}}\frac{d^2 a_L}{dR^2} + \left\{\frac{1}{2}\left[\left(E_g' + \frac{1}{2M_{12}}K_{gg}'\right) + \left(E_u' + \frac{1}{2M_{12}}K_{uu}'\right)\right] + \right.$$

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$$+\frac{1}{4M_{12}}\left(K'_{gu}+K'_{ug}\right)+\frac{L(L+1)}{2M_{12}R^2}+\frac{\Delta E}{2}\Big\}a_L+\Big\{\frac{1}{2}\Big[\Big(E'_g+\frac{1}{2M_{12}}K'_{gg}\Big)-\Big(E'_u+\frac{1}{2M_{12}}K'_{uu}\Big)\Big]-$$

$$-\frac{1}{4M_{12}}\left(K'_{gu}-K'_{ug}\right)\Big\}b_L+\frac{QR}{M_{12}}\frac{d}{dR}\left(\frac{b}{R}\right)=E'a_L$$

$$-\frac{1}{2M_{12}}\frac{d^2b_L}{dR^2}+\Big\{\frac{1}{2}\Big[\Big(E'_g+\frac{K'_{gg}}{2M_{12}}\Big)+\Big(E'_u+\frac{K'_{uu}}{2M_{12}}\Big)\Big]-\frac{1}{4M_{12}}\left(K'_{gu}-K'_{ug}\right)+\frac{L(L+1)}{2M_{12}R^2}-\frac{\Delta E}{2}\Big\}b_L+$$

$$+\Big\{\frac{1}{2}\Big[\Big(E'_g+\frac{K'_{gg}}{2M_{12}}\Big)-\Big(E'_u+\frac{K'_{uu}}{2M_{12}}\Big)\Big]+\frac{1}{4M_{12}}\left(K'_{gu}-K'_{ug}\right)\Big\}a_L-\frac{QR}{M_{12}}\frac{d}{dR}\left(\frac{a_L}{R}\right)=E'b_L$$

and the boundary condition for $R=0$

$$a(0)=b(0)=0 \tag{22}$$

Eqs. (21) for $R\to\infty$ are of the form:

$$-\frac{1}{2M_{12}}\frac{d^2a}{dR^2}=\left(E'-\frac{1}{2}\Delta E\right)\cdot a$$

$$-\frac{1}{2M_{12}}\cdot\frac{d^2b}{dR^2}=\left(E'+\frac{1}{2}\Delta E\right)\cdot b \tag{23}$$

Let $M_1 < M_2$ so that for definiteness $\Delta E>0$. Then three types of motion are possible depending on the value of E'.

a) $E'>\frac{1}{2}\Delta E$, i, e. the energy is higher than the K -level of lighter mesic atom. The meson may be near the nucleus M_1 as well as near nucleus M_2. If at first the meson was near the nucleus M_1 and there is charge exchange to the nucleus M_2 then it is necessary that the wave functions would obey the following condition: that $b(R)$ for $R\to\infty$ must include only a divergent wave

$$a(R)\approx C_1e^{i\varkappa_1R}+C_2e^{-i\varkappa_1R} \tag{24}$$

$$a(R) \approx C_3 e^{iK_2 R} + C_4 e^{-iK_2 R} \quad \text{where} \quad C_4 = 0 \tag{25}$$

b) $-\frac{1}{2}\Delta E < E' < \frac{1}{2}\Delta E$ i.e. the energy lies between separated mesic atoms. When $R \to \infty$ the meson cannot be near the separated lighter nucleus became of lack of energy. This case corresponds to the scattering of mesic atom with nucleus M_2 by the nucleus M_1 without the possibility of charge exchange. The wave functions should obey the condition in accordance to which the function $a(R)$ for $R \to \infty$ should not contain an exponentially increasing term:

$$a(R) \simeq D_1 e^{-\varkappa_1 R} + D_2 e^{\varkappa_1 R}$$

$$D_2 = 0$$

$$b(R) \simeq D_3 e^{-iK_2 R} + D_4 e^{iK_2 R} \tag{26}$$

$$\varkappa_1^2 = -K_1^2 = -2M\left(E' - \frac{1}{2}\Delta E\right) \tag{27}$$

It is obvious that the conditions (22) and (26) (as well as the conditions (22) and (24)) can be satisfied at any energy, taken from the considered energy region and determine the solution of the system of equations (21) with the accuracy up to the normalization.

c) $E' < -\frac{1}{2}\Delta E$ is a region of the discret spectrum corresponding to the bound states of mesic molecules. For $R \to \infty$ two conditions are imposed to the solution of the system: the absence of the increasing exponents in both functions $a(R)$ and $b(R)$

$$a(R) \approx Z_1 e^{-\varkappa_1 R} + \mathcal{T}_2 e^{\varkappa_1 R}$$

$$b(R) \approx Z_3 e^{-\varkappa_2 R} + \mathcal{T}_4 e^{\varkappa_2 R} \tag{28}$$

$$\mathcal{T}_2(E') = 0, \qquad \mathcal{T}_4(E') = 0 \tag{29}$$

$$\varkappa_1^2 = 2M_{12}\left(|E'| + \frac{1}{2}\Delta E\right); \qquad \varkappa_2^2 = 2M_{12}\left(|E'| - \frac{1}{2}\Delta E\right)$$

The conditions (28) and (22) may be satisfied only for definite values of E' ,being energy levels of the mesic molecule. In order to find solutions satisfying conditions (22) for $R = 0$ and asymptotic conditions of the type (24), (26) and 28) when $R \to \infty$ we can find two linearly independent solutions satisfying the condition(22) and construct a linear combination satisfying asymptotic conditions for $R \to \infty$. Note that between two any solutions of the system of linear equations of the second order (16) there exists a connection which can be easily established if we take into account that

$$K'_{gu} - K'_{ug} = -2\,\mathrm{div}\,Q \qquad (30)$$

Indeed, if $\{\varphi_1, H_1\}$ and $\{\varphi_2, H_2\}$ are the solutions of the system (16), then:

$$\mathrm{div}\left\{(\varphi_2 \nabla \varphi_1 - \varphi_1 \nabla \varphi_2) + (H_2 \nabla H_1 - H_1 \nabla H_2) + 2Q\,(H_1 \varphi_2 - H_2 \varphi_1)\right\} = 0 \qquad (31)$$

For the functions (20), satisfying the condition (22) the indentity (31) takes the form:

$$\left(a_2 \frac{da_1}{dR} - a_1 \frac{da_2}{dR}\right) + \left(b_2 \frac{db_1}{dR} - b_1 \frac{db_2}{dR}\right) + 2Q(a_1 b_2 - a_2 b_1) = 0 \qquad (32)$$

The relation (32) may be used for testing the corrections of the numerical integration. As linearly independent solutions for the Eqs. (21) we may choose, for example, the solutions determined for R = 0* by the conditions:

$$\begin{cases} a(0) = b(0) = 0 \\ a'(0) = b'(0) = 1 \end{cases} \qquad \begin{matrix} g = h = 0 \\ g' = \sqrt{2}\,; \; h' = 0 \end{matrix} \qquad (33)$$

$$\begin{cases} a(0) = b(0) = 0 \\ a'(0) = -b'(0) = 1 \end{cases} \qquad \begin{matrix} g = h = 0 \\ g' = 0; \; h' = \sqrt{2} \end{matrix} \qquad (34)$$

CHARGE EXCHANGE CROSS SECTION

Let in the energy region $E' > \frac{1}{2}\Delta E$ the solutions (20) determined by the conditions (24) and (25) respectively be of the form

---

* Since the functions $a(R)$ and $b(R)$ vanish exponentially inside the potential barriers for R→0 the conditions (33), (34) may be given for numerical integration without essential error for some small $R_0 \neq 0$. In our paper $R_0 = 0{,}2$. This corresponds to the replacement of potentials $E_g + \frac{1}{2M_{\mu}} K_{gg}$ and $E_u + \frac{1}{2M_{\mu}} K_{uu}$ by the infinite wall at $R = R_0$.

$$\mathrm{I.}\begin{cases} a_L^{(1)} \approx a_0^{(1)} \sin\left(\kappa_1 R - \frac{\pi L}{2} + \gamma_1\right) \\ b_L^{(1)} \approx b_0^{(1)} \sin\left(\kappa_2 R - \frac{\pi L}{2} + \delta_1\right) \end{cases} \qquad \mathrm{II.}\begin{cases} a_L^{(2)} \approx a_0^{(2)} \sin\left(\kappa_1 R - \frac{\pi L}{2} + \gamma_2\right) \\ b_L^{(2)} \approx b_0^{(2)} \sin\left(\kappa_2 R - \frac{\pi L}{2} + \delta_2\right) \end{cases} \tag{35}$$

where constants $a_0^{(i)}$, $b_0^{(i)}$ are determined by numerical integration of (21). According to (32) there is a connection between the coefficients and the phases of (35)

$$\kappa_1 a_0^{(1)} a_0^{(2)} \sin\gamma_{21} + \kappa_2 b_0^{(1)} b_0^{(2)} \sin\delta_{12} = 0 \tag{36}$$

where

$$\gamma_{21} = \gamma_2 - \gamma_1 ; \qquad \delta_{21} = \delta_2 - \delta_1 \tag{37}$$

Forming a linear combination from (35) which satisfies the conditions (33), (34) and the normalization for $R \to \infty$ we obtain

$$a \approx \frac{\alpha e^{i\left(\kappa_1 R - \frac{\pi L}{2}\right)} - e^{-i\left(\kappa_1 R - \frac{\pi L}{2}\right)}}{2i\kappa} ;$$

$$b = -\frac{N_1 N_2 \sin\delta}{\left(N_2 e^{-i\gamma_1} - N_1 e^{-i\gamma_2 + i\delta}\right)\kappa_1} \cdot e^{i\left(\kappa_2 R - \frac{\pi L}{2} + \delta_2\right)} \tag{38}$$

where

$$N_1 = \frac{b_1^0}{a_1^0} ; \quad N_2 = \frac{b_2^0}{a_2^0} ; \qquad \alpha = \frac{N_2 e^{i\gamma_1} - N_1 e^{i\gamma_2 + i\delta}}{N_2 e^{-i\gamma_1} - N_1 e^{-i\gamma_2}} ; \tag{39}$$

In accordance with the general theory of inelastic collisions the effective charge exchange cross section corresponding to the partial wave L :

$$\sigma_{L\,ex.} = \frac{\pi}{k^2}(2L+1)\left(1 - |\alpha|^2\right). \tag{40}$$

$$= 4\pi\alpha^2(2L+1)\cdot\frac{N_1^2 N_2^2 \sin^2\delta_{21}}{N_1^2+N_2^2-2N_1N_2\cdot\cos(\xi_{21}-\delta_{21})}\cdot\frac{k_2}{k_1^3} \qquad (40)$$

and the elastic scattering cross section:

$$\sigma_{el} = \frac{\pi}{k_1^2}(2L+1)|1-\alpha|^2 = 4\pi(2L+1)\cdot$$

[redacted]

$$\frac{[redacted]}{N_1^2+N_2^2-2N_1N_2\cos(\xi_{21}-\delta_{21})}$$

Since the collisions occur at a very low energy the scattering in the S state is the most essential. In the region $1 \ll R \ll \frac{1}{k_1}$, the solutions (35) for $L = 0$ take the form :

$$\text{I}\begin{cases} a_0^{(1)}(R) = C_1^{(1)}R + C_2^{(1)} \\ b_0^{(1)} = C_3^{(1)}\sin(k_2^0 R + \delta_2^{(1)}) \end{cases} \qquad \text{II}\begin{cases} a_0^{(2)} = C_1^{(2)}R + C_2^{(2)} \\ b_0^{(2)} = C_3^{(2)}\sin(k_2^0 R + \delta_2^{(2)}) \end{cases} \qquad (42)$$

$$k_2^0 = \sqrt{2M\Delta E}$$

and the condition (36) gives:

$$C_1^{(1)}C_2^{(2)} - C_2^{(1)}C_1^{(2)} + k_2^0 C_3^{(1)} C_3^{(2)}\sin(\delta_2^{(2)} - \delta_1^{(1)}) = 0$$

The linear combination (38) which satisfies the condition (42) and accordingly normalized has the form:

$$a = R + \frac{\beta_1\gamma_2 - \gamma_1\beta_2 e^{i\delta_{21}}}{\gamma_2 - \gamma_1 e^{i\delta_{21}}} \qquad b = -\frac{\gamma_1\gamma_2\sin\delta_{21}}{\gamma_2-\gamma_1 e^{i\delta_{21}}}\, e^{i(k_2^0 R + \delta_2)} \qquad (43)$$

where

$$\beta_i = \frac{C_2^{(i)}}{C_1^{(i)}};\quad \gamma_i = \frac{C_3^{(i)}}{C_1^{(i)}};\qquad \delta_{21} = \delta_2 - \delta_1$$

The charge exchange cross section is:

$$\sigma_{ex} = 4\pi\frac{k_2^0}{k_1}\cdot\frac{\gamma_1^2\gamma_2^2 \sin^2\delta_{21}}{\gamma_1^2+\gamma_2^2-2\gamma_1\gamma_2\cos\delta_{21}} = 4\pi f a_\mu^2\frac{v_0}{v} \qquad (44)$$

and the elastic scattering cross section is

$$\sigma_{el} = 4\pi\frac{\beta_1^2\gamma_2^2+\gamma_1^2\beta_2^2-2\gamma_1\gamma_2\beta_1\beta_2\cdot\cos\delta_{21}}{\gamma_1^2+\gamma_2^2-2\gamma_1\gamma_2\cos\delta_{21}}\, a_\mu^2 \qquad (45)$$

If v_1 is the relative velocity of particles, N_2 – the number of nuclei of the isotope with mass M_2 then the probability of the charge exchange is

$$W = N_2 \sigma_{ex} v_1 = 4\pi v_2^0 \frac{\sigma_1^2 \sigma_2^2 \sin^2 \delta_{21}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1 \sigma_2 \cos \delta_{21}} a_\mu^2 N_2 \tag{46}$$

$$v_2^0 = \sqrt{\frac{2\Delta E}{M}}; \qquad a_\mu = \frac{\hbar^2}{m_\mu e^2};$$

The values of $v, \sigma_{ex}; f$ and σ_{el} for the systems of proton-deuteron, system proton-tritium and deuteron-tritium are given in Table 1.

SCATTERING OF HEAVY MESIC ATOMS BY LIGHT ISOTOPE NUCLEI, CROSS SECTIONS AND WAVE FUNCTIONS

In the energy region $-\frac{1}{2}\Delta E \le E' \le \frac{1}{2}\Delta E$ when the concentration of the heavy hydrogen isotope is small the most essential process is an elastic scattering of heavy mesic atoms by light isotope nuclei and further after mesic atom slowing down -formation of mesic molecules.

Let the solutions obtained by numerical integration of (21) with boundary conditions (33) and (34) respectively for $R \to \infty$ take the form:

$$a_L^{(1)}(R) \approx d_{L_1}^{(1)} e^{-\varkappa_1 R} + d_{L_2}^{(1)} e^{\varkappa_1 R}; \qquad a_L^{(2)}(R) \approx d_{L_1}^{(2)} e^{-\varkappa_1 R} + d_{L_2}^{(2)} e^{\varkappa_1 R}$$

$$b_L^{(1)}(R) \approx d_{L_3}^{(1)} \sin\left(k_2 R - \frac{\pi L}{2} + \omega^{(1)}\right); \quad b_L^{(2)}(R) \approx d_{L_3}^{(2)} \sin\left(k_2 R - \frac{\pi L}{2} + \omega^{(2)}\right) \tag{47}$$

Forming the linear combination from (47) satisfying the condition (26) we obtain within the accuracy of constant factor

$$b_L(R) \approx \sin\left(k_2 R - \frac{\pi L}{2} + \omega\right) \qquad a_L(R) \approx \frac{d_{L_1}^{(1)} d_{L_2}^{(2)} - d_{L_2}^{(1)} d_{L_1}^{(2)}}{T} e^{-\varkappa_1 R} \tag{48}$$

where

$$tg\,\omega = \frac{d_{L_3}^{(1)} \cdot d_{L_2}^{(2)} \sin \omega^{(1)} - d_{L_3}^{(2)} d_{L_2}^{(1)} \sin \omega^{(2)}}{d_{L_3}^{(1)} \cdot d_{L_2}^{(2)} \cos \omega^{(1)} - d_{L_3}^{(2)} d_{L_2}^{(1)} \cdot \cos \omega^{(2)}}; \tag{49}$$

Partial cross section corresponding to the L wave is

$$\sigma = \frac{4\pi}{k^2}(2L+1)\sin^2\omega = \frac{4\pi}{k^2}(2L+1)\,\frac{\left[d_{L_2}^{(1)} d_{L_2}^{(2)} \sin\omega^{(1)} - d_{L_2}^{(2)} d_{L_2}^{(1)} \sin\omega^{(2)}\right]^2}{T^2} \qquad (50)$$

It is very important to know wave functions and effective scattering cross section when the kinetic energy of mesic atom is small ($k_2 \ll 1$). In the region $R_0 \ll R \ll \frac{1}{k_2}$ the solutions (4?) for the wave may be represented in the form:

$$a_0^{(1)} \approx d_1^{(1)} e^{-\varkappa_1^0 R} + d_2^{(1)} e^{\varkappa_1^0 R}$$

$$b_0^{(1)} \approx R + D^{(1)} \qquad \varkappa_1^0 = \sqrt{2M\Delta E} \qquad (51)$$

The linear combination (51) satisfying the condition (26) in the region $R_0 \ll R \ll \frac{1}{k_2}$ is of the form:

$$a \approx \frac{D^{(2)} - D^{(1)}}{2(d_2^{(2)} - d_2^{(1)})} e^{-\varkappa_1^0 R} \qquad b \approx R + \frac{D^{(1)} d_2^{(2)} - D^{(2)} d_2^{(1)}}{d_2^{(2)} - d_2^{(1)}}; \qquad (52)$$

The normalization of the functions (52) is chosen so that for $R \to \infty$ it corresponds to a plane wave of the relative motion of nuclei (with coefficient 1) and to a meson that is near a heavier nucleus. In Table V the wave functions $a(R)$ and $b(R)$ for the systems proton-deuteron, proton-tritium and deuteron-tritium are given. The effective cross section of scattering of the mesic atom with a heavier isotope by nuclei of lighter ones at small kinetic energy of mesic atom is:

$$\sigma = 4\pi \left| \frac{D^{(1)} d_2^{(2)} - D^{(2)} d_2^{(1)}}{d_2^{(2)} - d_2^{(1)}} \right|^2 = 4\pi\lambda^2 a_\mu^2 \qquad (53)$$

The values for σ and λ are given in Table II. Note that in the case of the process $d\mu + p \longrightarrow d\mu + p$ σ elastic is not anomalously small as it is indicated in /11/. Free path ℓ of the atom $d\mu$ which was produced as a result of the charge exchange may be determined by means of the formula

$$\ell = \frac{1}{\sigma N \xi_{tr}} \ln \frac{E_1}{E_2} \qquad (54)$$

where $K = \frac{2M_{12}}{M_1 + M_2} = \frac{4}{9}$ is the mean value of the energy transmitted in $d_\mu + p$ collision $N = 4 \cdot 10^{22}$ $E = 45\ ev$ is the energy acquired by the mesic atom d_μ while charge exchanging $E_2 \sim 2 \cdot 10^{-4} ev$ is the final energy. (In this case we make the rough approximation that d_μ is believed to move along a straight line since the deviation of d_μ in collision with proton can not exceed 30^0 in the laboratory system of coordinates). The value of the free path according to (54) is $\ell \sim 0.1$ mm.(The free path owing to the diffusion of d_μ with the thermal energy is also of order of 0.1 mm) which is applying less than the experimental 'hole' of about 1 mm.

MESIC MOLECULE ENERGY LEVELS

Let in the energy region $E' < -\frac{1}{2}\Delta E$ the solutions of Eq. (21) under the initial conditions (33) and (34) correspondingly have for $R \to \infty$ the following form:

$$a_i(R) \approx \mathcal{F}_1^{(i)}(E')e^{-\mathscr{X}_1 R} + \mathcal{F}_2^{(i)}(E')e^{\mathscr{X}_1 R}$$

$$b_i(R) \approx \mathcal{F}_3^{(i)}(E')e^{-\mathscr{X}_2 R} + \mathcal{F}_4^{(i)}(E')e^{\mathscr{X}_2 R} \qquad (55)$$

Then, forming the linear combination the increasing exponents can be excluded only under the condition:

$$\begin{vmatrix} \mathcal{F}_2^{(1)}(E') & \mathcal{F}_2^{(2)}(E') \\ \mathcal{F}_4^{(1)}(E') & \mathcal{F}_4^{2}(E') \end{vmatrix} = 0 \qquad (56)$$

The condition (56) determines mesic molecule energy levels. By means of the numerical integration of the system (21) for various E' we can choose the values of E' satisfying the condition (56). The mesic molecule energy levels obtained in such a way are given in Table III. In Fig. 1 the values of the functions a (R) and b (R) are given for the bound state of the mesic molecule (pt)$_\mu$ normalized by the condition

$$\int_0^\infty (|a(R)|^2 + |b(R)|^2 dR = 1 \qquad (57)$$

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ENERGY LEVELS OF MESIC MOLECULES WITH IDENTICAL NUCLEI

As we have already pointed out the system for the mesic molecules with identical nuclei breaks up two independent equations. The effective potentials of interaction with corrections taking into account the motion of nuclei* $E_g' + \frac{1}{2M_\mu} K_{gg}'$ are approximated with a good accuracy by the well-known Morse function:

$$U = A\left[e^{-2\alpha(R-R_0)} - 2e^{-\alpha(R-R_0)}\right] \qquad (58)$$

The values of the effective potentials of interaction are given in /13/. The deviations from the Morse function from the true values of effective potentials of interaction can not change considerably the value of the levels since these deviations are appreciable only in those regions (for very large or very small R) where the wave functions decrease exponentially. The values of the energy levels are given in Table III.

SCATTERING OF MESIC ATOMS BY NUCLEI IDENTICAL TO THE MESIC ATOM NUCLEUS

For small energy of the relative motion of ($\varkappa R \ll 1$) it is not difficult to calculate effective cross section of mesic atom scattering by nuclei of the isotope**. For $E' > 0$ the solution of the Schredinger equation with potential (58) is of the form

$$g = e^{-\xi/2} i\left[\xi^{i\varkappa} e^{-i\varphi} F\left(-\frac{\sqrt{2MA}}{\alpha} + \frac{1}{2} + i\varkappa; 1+2i\varkappa; \xi\right) - \xi^{i\varkappa} e^{i\varphi} F\left(-\frac{\sqrt{2MA}}{\alpha} + \frac{1}{2} - i\varkappa; 1-2i\varkappa; \xi\right)\right] \qquad (59)$$

* Compare with /16/, where the approximation was performed without taking into account the corrections to the motion of nuclei.

** The energy of the relative motion must be considerably higher than the energy of the superfine splitting in the mesic atom (in the case of the mesic atom $p\mu$ it is about of 0.2 ev). The effects at lower energy are considered in /6/.

where

$$\xi = \frac{2\sqrt{MA}}{\alpha} e^{-\alpha R}; \quad \varkappa = \frac{\sqrt{2ME'}}{\alpha} = \frac{k}{\alpha}; \quad e^{i\varphi} = \frac{\Gamma(1+2i\varkappa)\,\Gamma\left(-\frac{\sqrt{2MA}}{\alpha}+\frac{1}{2}-i\varkappa\right)}{\Gamma(1-2i\varkappa)\,\Gamma\left(-\frac{\sqrt{2MA}}{\alpha}+\frac{1}{2}+i\varkappa\right)}$$

For $R \to \infty\ (\xi \to 0)$ this solution equals asymptotically:

$$g \approx \sin\left(kR - \frac{k \ln 2\frac{\sqrt{2MA}}{\alpha}}{\alpha} - kR_0 + \varphi\right) \tag{60}$$

If the energy of the relative motion is rather small $(kR_0 \ll 1)$ then, in the region $R_0 \ll R \ll 1/k$

$$g \sim R - \lambda_g \tag{61}$$

The scattering length may be obtained from (64),(65)

$$\lambda_g = \left\{\psi\left(-\frac{\sqrt{2MA}}{\alpha}+\frac{1}{2}\right) - 2\psi(1) + \ln\frac{2\sqrt{2MA}}{\alpha}\right\}\frac{\alpha}{2MA} + R_0 \tag{62}$$

where $\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$; . Note that if the value $\left(\frac{\sqrt{2MA}}{\alpha} - \frac{1}{2}\right)$ is close to the integer number this corresponds to the fact that the mesic molecule has a level (real or virtual with the energy close to zero) then, due to the fact that the function $\psi(x)$ has poles for integer negative numbers x the value of λ_g may be very large. The possibility of such a resonance has been observed in /2/.

The solution of the second equation (21) may be also obtained easily if the potential $V_u = \left(E'_u(R) + \frac{1}{2M_u} K'_{uu}(R)\right)$ is approximated by an exponent:

$$V_u = Be^{-\beta R} \tag{63}$$

this may be made with a good accuracy for the values of R essential for the scattering. The Schrödinger equation with potential (63) reduces to the Bessel equation of the imaginary argument by introducing a new variable quantity. Thus, for E' = 0

$$h(R) = \frac{2}{\beta} K_0\left(\frac{2\sqrt{2MB}}{\beta} e^{-\beta R/2}\right)$$

where $K_0(t)$ is the Hankel function of the imaginary argument. The normalization of the function $h(R)$ is chosen so that for $R \to \infty$ the solution will be of the form:

$$h(R) \approx R - \lambda_u \tag{64}$$

(taking into account that for $t \to 0,\ K_0(t) \to \ln 2/t - C$ where C = 0.577 is the Euler constant) we obtain

$$\lambda_u = 2/\beta \left[C + \ln \frac{\sqrt{2MB}}{\beta} \right]. \tag{65}$$

Taking into account that the mesonic function $\Sigma_g(\vec{r}, R)$ is symmetrical with respect to the nucleus exchange and $\Sigma_u(\vec{r}, R)$ is antisymmetrical, it may be concluded that in the S- wave the relative motion of identical nuclei will be described by the function $g(R)$, if the summary spin of nuclei is even and by the function $h(R)$, if the summary spin of nuclei is odd. The effective cross section of the hydrogen mesic atom scattering by protons (and mesic atoms of tritium by tritium nuclei) at small energies is to be of the form:

$$\sigma = 2\pi\left(\frac{1}{4} \frac{\lambda_g^2}{1+K^2\lambda_g^2} + \frac{3}{4} \frac{\lambda_u^2}{1+K^2\lambda_u^2} \right) \tag{66}$$

and for the scattering of mesic atoms of deuterium by deuterons:

$$\sigma = 2\pi\left(\frac{2}{3} \frac{\lambda_g^2}{1+K^2\lambda_g^2} + \frac{1}{3} \frac{\lambda_u^2}{1+K^2\lambda_u^2} \right). \tag{67}$$

The formula (66) is analogous to that describing the scattering of neutrons by protons. For large λ_g in accordance with (66) there will be a resonance in the scattering when $K \to 0$. (The terms $K^2\lambda_u^2$ in (66) are written for better analogy with the well-known deuteron formula, the consideration of these terms represents essentially the exceeding of the accuracy since λ_u corresponds to the repulsion and must coincide in magnitude with the radius of action of forces: $\lambda_u \approx R_0$). This takes place just in the scattering $p_\mu + p \to p_\mu + p$ since the mesic molecule $(pp)_\mu$ have a virtual level with the energy close to 0. The effective scattering cross section $p_\mu + p$ calculated according to (66) coincides well with the cross section given in /7/. For the scattering $d_\mu + d$ the cross section calculated according to (67) turns out to be twice the value given in /7/.

MESIC MOLECULE FORMATION

Moving in the matter the hydrogen mesic atoms in virtue of their neutrality can go easily through the electronic shells of the hydrogen molecules and approaching the nuclei they can form mesic molecules (more exactly, mesic molecular ions) $(pp)_\mu^+$, $(pd)_\mu^+$ etc. (like the well-known molecular ions H_2^+, HD^+ etc.). The binding energy of the mesic molecule, in principle, may be transmitted to the radiation, electron shell and to the nucleus coupled by

chemical forces with that which forms the mesic molecule.

The last transition, however, may have some meaning only in the case when the mesic molecule forms itself in the state with very small coupling energy (of order of the coupling energy of the ordinary molecule). As we can see from Table III this condition can not be fulfilled for any of the molecules (perhaps, except for $(dd)_\mu^+$). Since the size of the mesic molecule is less than the atomic one the relations between the probabilities of formation of a mesic molecule in a radiation way and by means of the recoil of the energy to the electron of the shell may be expressed using the standard theory of intrinsic conversion of electrons under nuclear transitions. In the considered region of transmitted energies (tens -hundreds of ev) the coefficients of intrinsic conversion are very large; therefore the probability of the radiative formation of mesic molecules is less incomparably than the conversional one. Because the formation of mesic molecules takes place at small relative energies of mesic atoms an electric dipole transition from the S-wave of continuous spectrum with the conversion on electron will be the most important one. Let ρ_1, ρ_2, ρ_μ be distances from the nuclei M_1, M_2 and μ^- meson respectively from an arbitrary point; then the Coulomb field of the system at large distances has the form

$$\frac{e}{\rho_1}+\frac{e}{\rho_2}-\frac{e}{\rho_\mu}\approx\frac{e}{\rho}+\frac{d\rho}{\rho^3} \tag{68}$$

where ρ is the distance from the center of mass of nuclei, and d is the dipole momentum of the system

$$\vec{d}=-\frac{e}{2}\frac{M_2-M_1}{M_2+M_1}\vec{R}-\frac{e}{2}(\vec{r}_1+\vec{r}_2)$$

$$\left(\vec{R}=\vec{R}_2-\vec{R}_1;\quad \vec{r}_1=\vec{r}-\vec{R}_1;\quad \vec{r}_2=\vec{r}-\vec{R}_2\right). \tag{69}$$

If the precise Coulomb functions of the hydrogen atom are taken as wave functions of electron then in analogy with the probability of conversion under the nuclear transition we obtain for the probability of mesic molecule formation by means of dipole conversion:

$$W=\frac{16}{3}\frac{1}{\hbar a_0}\cdot\frac{2\pi\eta^2 e^{-4\eta\,\mathrm{arcctg}\,\eta}\cdot N}{(1+\eta^2)(1-e^{-2\pi\eta})}\sum_{M_b}\left|\langle d\rangle\right|^2. \tag{70}$$

Here $a_0=\frac{\hbar^2}{m_0e^2}$; $\eta=\frac{e^2}{\hbar v_0}$

(m_0, v_0 are mass and velocity), N is the number of nuclei in cm^3, $\langle d\rangle$ is the matrix [illegible] of the dipole [illegible] between the wave functions of the mesic molecule related to the

continuous and the discrete spectrum respectively.

$$\langle d \rangle = \int \Psi_{mol}^{f*}\, d\, \Psi_{mol}^{i}\, (dR)(d\tau_\mu) \qquad (71)$$

The integration in (71) is performed over the coordinates of μ^--meson and of nuclei). The integration over the mesonic coordinates in (71) leads to the integral:

$$d_{gu} = \int \Sigma_g \tfrac{1}{2}(\tau_1 + \tau_2)\Sigma_u (d\tau_\mu) \qquad (72)$$

From the reasons of symmetry it is clear that d_{gu} is directed along R: $d_{gu} = \gamma(R)R$, where $\gamma(R)$ is a function R. In the approximation (LCAO)

$$\gamma(R) = -\frac{1}{2}\frac{1}{\sqrt{1-e^{-2R}(1+R+R^2/3)^2}} \approx -\frac{1}{2}, \text{ for } R > 3 \qquad (73)$$

The approximation LCAO fails for small R, however, it is clear that since the function $\gamma(R)$ corresponding to the repulsion state Σ_u for small values of R vanishes exponentially the small values of R contribute insignificantly to the matrix element $\langle d \rangle$. Note that the approximation $\gamma = -1/2$ is well fulfilled for small R too, if the calculation of d_{gu} is made with the precise functions Σ_g, Σ_u. The summation in (70) is made over all possible final states of the mesic molecule. In calculating the normalization of the wave function of the continuous spectrum is assumed to be chosen so that at the infinity there a plane wavewith the coefficient one, and the wave function of the discrete spectrum is normalized to unity. In this case the matrix element $\langle d \rangle$ will have the dimension of $a_\mu^{5/2}$ ($a_\mu = \hbar^2/m_\mu e^2$) Separating out dimension factor it is convenient to rewrite the formula (70) in the form

$$W = \frac{16}{3}(N a_e^3)\left(\frac{m_e}{m_\mu}\right)^5 \cdot \frac{e^2}{\hbar a_e}\frac{2\pi\eta^2 e^{-4\eta\,\mathrm{arcctg}\,\eta}}{(1+\eta^2)(1-e^{-2\pi\eta})}\left\{\sum_{n_f}\left|\langle d \rangle\right|^2\right\}, \qquad (74)$$

where the matrix element $\langle d \rangle$ is calculated in dimensionless ('mesic atom') units. It is easy to see that the first term in the dipole momentum (69) has non-zero matrix elements only for transitions with mesonic functions of identical parity $\Sigma_g \rightarrow \Sigma_g$; $\Sigma_u \rightarrow \Sigma_u$ while the second term gives transitions only between mesonic functions of different parity ($\Sigma_u \rightarrow \Sigma_g$; $\Sigma_g \rightarrow \Sigma_u$). The dipole transitions from the S-wave of the continuous spectrum may occur only into the rotation state with $L = 1$. From the table III one can see that for all mesic molecules (except $/tt/_\mu$) the transition is performed into the ground rotation state ($L = 1$; $v = 0$). For mesic molecules $/tt/_\mu$) the transition into the vibration-rotational state is possible too.

The probability of formation of mesic molecules with identical nuclei $L = 1$; $n = 1$ may be calculated analogously as it was made for mesic molecules in $(dd)_\mu$. The probabilities of formation of these molecules are given in the Table 4. In the case of mesic molecules with different nuclei of greatest interest is the calculation of probability of mesic molecule formation when the collision of the mesic atom of heavier isotope with lighter nucleus occurs, for example $d_\mu + p \to (dp)_\mu$ (in the collision $p_\mu + d$ the charge exchange is the most probable process $p_\mu + d \to d_\mu + p$). The wave function of the initial state has the form:

$$\Psi^{(i)}_{[illegible]} = \mathcal{G}_0(R)\Sigma_g + H_0(R)\Sigma_u , \qquad (75)$$

where the functions $\mathcal{G}_0$, $\mathcal{H}_0$ are connected with the functions $a_0(R)$, $l_0(R)$ by the relation [illegible]. The wave function of the final state corresponding to the rotation level of the mesic molecule with $L = [illegible]$ has the form

$$\Psi^{(f)} = \left\{ [illegible] \Sigma_g - L_1(R)\Sigma_u \right\} \cdot \frac{1}{R} Y_{1M}(\theta, \varphi) . \qquad (76)$$

[illegible] over the angle and the summation over M, we get in mesic atom units

$$[illegible] \int \left[[illegible] a(R) - [illegible] l_1(R) \right] R \, dR . \qquad ([illegible])$$

[illegible] are calculated numerically. The probability of formation of mesic molecules $[illegible]$ found in accordance with (77) and (78) are given in the

APPENDIX

The functions [illegible] and $l_{uu}(R)$ are calculated in /[illegible]/ by means of precise wave functions $\Sigma_{[illegible]}$ [illegible]. We give for comparison the values $l_{gg}(R)$ and $l_{uu}(R)$ [illegible] LCAO consistent for large R.

$$\Sigma_g = [2\pi(1+S)]^{-1/2} (e^{-r_1} + e^{-r_2})$$

$$\Sigma_u = [2\pi(1-S)]^{-1/2} (e^{-r_1} - e^{-r_2}) \quad (LCAO) \qquad D.1.$$

$$S = (1 + R + R^2/3) e^{-R}$$

D.2.

[illegible] ($[illegible]$) consistent for small R, when Σ_g and Σ_u transform res-

pectively into 1s and 2p levels of He.

$$\Sigma_g = \frac{1}{N_g} e^{-(r_1 + r_2)}$$

$$\Sigma_u = -\frac{1}{N_u} e^{-1/2(r_1 + r_2)} (r_1 \cos\theta_1 - r_2 \cos\theta) \qquad \text{D.3}$$

$$N_g^2 = 3/8 \, e^{-2R} \left(1 + 2R + 3/4 R^2\right) \qquad \text{D.4}$$

$$N_u^2 = 4\pi \left(1 + R + \frac{9}{20} R^2 + \frac{7}{60} R^3 + \frac{1}{60} R^4\right) e^{-R}. \qquad \text{D.4'}$$

In the approximation (LCAO):

$$K_{gg} = \frac{1}{2} - \frac{S}{2(1+S)} - \frac{1}{36} \frac{R^2(1+R)^2}{(1-S)^2} \cdot e^{-2R} \qquad \text{D.5}$$

$$K_{uu} = \frac{1}{2} + \frac{S}{2(1-S)} - \frac{1}{36} \frac{R^2(1+R)^2}{(1-S)^2} \cdot e^{-2R}. \qquad \text{D.5'}$$

In the approximation (U . A):

$$K_{gg} = 1 - \left(\frac{N_g'}{N_g}\right)^2 \approx_{R \to 0} 1 - \frac{16}{9} R^2 \qquad \text{D.6}$$

$$K_{uu} = 2/R^2 + 1/4 - \left(\frac{N_u'}{N_u}\right)^2 \approx_{R \to 0} \frac{2}{R^2} + \frac{1}{4}. \qquad \text{D.6'}$$

For comparison in Fig. 2,3 are given the values K gg and K uu , calculated in [15] and according to the approximation (LCAO) and (U . A). For the value Q(R) we get:

$$(LCAO)\; Q = -\frac{M_2 - M_1}{M_2 + M_1} \cdot \frac{R(R+1)e^{-R}}{6\sqrt{1 - S^2}}$$

$$(UA)\,Q = -\frac{M_2-M_1}{M_2+M_1}\cdot\frac{16\pi}{81}\cdot\frac{e^{-3R/2}}{N_g N_u}\left(1+\tfrac{3}{2}R+\tfrac{3}{4}\cdot R^2\right).$$

We note that Q calculated in accordance with (LCAO) for $R \to 0$ coincide well with the values of Q, calculated in accordance with

$$(LCAO)\begin{cases} K_{gu} = \dfrac{1}{2\sqrt{1-S^2}}\left\{1+e^{-R}\left(1+R-\dfrac{R^2}{3}\right)-\dfrac{R^2(1+R)^2}{9(1-S)}e^{-2R}\right\}\dfrac{M_2-M_1}{M_2+M_1} \\ K_{ug} = \dfrac{1}{2\sqrt{1-S^2}}\left\{1-e^{-R}\left(1+R-\dfrac{R^2}{3}\right)-\right. \end{cases}$$

$$\left.-\frac{R^2(1+R)^2}{-9(1+S)}e^{-2R}\right\}\frac{M_2-M_1}{M_2+M_1}.$$

Thus, in the all region R the approximation $Q = Q_{LCAO}$ may be taken.

The quantities $K_{gu}(R)$ and $K_{ug}(R)$ in the approximation (LCAO) and (U A) equal cor- [illegible]

$$(UA)\begin{cases} K_{gu} = \dfrac{M_2-M_1}{M_2+M_1}\cdot\dfrac{64\pi}{81}\cdot\dfrac{e^{-3R/2}}{N_g N_u}\left(1+\tfrac{3}{2}R+\tfrac{3}{4}R^2\right)\left(1-\dfrac{R}{2}\dfrac{N_u'}{N_u}\right)\dfrac{1}{R} \\ K_{ug} = \dfrac{M_2-M_1}{M_2+M_1}\cdot\dfrac{64\pi}{81}\cdot\dfrac{e^{-3R/2}}{N_g N_u}\left\{\dfrac{1}{2}\dfrac{N_g'}{N_g}\left(1+\tfrac{3}{2}R+\tfrac{3}{4}R^2\right)+\right. \end{cases}$$

$$\left.+\frac{3}{8}R+\frac{9}{16}R^2\right).$$

Show the behaviour of the function for $R \to 0$.

$$K_{gu}(LCAO) \to \frac{M_2-M_1}{M_2+M_1}\cdot\frac{2}{\sqrt{3}\,R} \approx 1{,}15\,\frac{1}{R}\,\frac{M_2-M_1}{M_2+M_1};$$

$$K_{gu}(UA) \to \frac{M_2-M_1}{M_2+M_1}\cdot\frac{64\sqrt{2}}{81R} \approx 1{,}12\,\frac{1}{R}\,\frac{M_2-M_1}{M_2+M_1}$$

$$K_{ug}(LCAO) \to \frac{M_2-M_1}{M_2+M_1}\cdot\frac{7\sqrt{3}}{18}R \approx 0{,}67\cdot R\,\frac{M_2-M_1}{M_2+M_1}$$

$$K_{ug}(UA) \to \frac{M_2-M_1}{M_2+M_1}\cdot\frac{8\sqrt{2}}{35}R \approx 0{,}322R\,\frac{M_2-M_1}{M_2+M_1}$$

T a b l e I

charge exchange cross section

	$p_\mu + d \to d_\mu + p$	$p_\mu + t \to t_\mu + p$	$d_\mu + t \to t_\mu + d$
f	2.1	0.84	0.0067
$\sigma_{ex} \cdot V$	$3.42 \cdot 10^{-13} \frac{cm^3}{sec}$	$1.49 \cdot 10^{-13} \frac{cm^3}{sec}$	$1.15 \cdot 10^{-15} \frac{cm^3}{sec}$
σ_{ex}	$1.98 \cdot 10^{-19}\ cm^2$	$1.53 \cdot 10^{19}\ cm^2$	$2.41 \cdot 10^{-19}\ cm^3$

T a b l e II

mesic atom elastic scattering cross section

	$d_\mu + p \to d_\mu + p$	$t_\mu + p \to t_\mu + p$	$t_\mu + d \to t_\mu + d$
λ	2.03	2.66*	6.7
$\sigma_{el}(0)$	$3.39 \cdot 10^{-20}\ cm^2$	$5.84 \cdot 10^{-20}\ cm^2$**	$36.9 \cdot 10^{-20}\ cm^2$

T a b l e III

mesic molecule levels (ev) (for mesic molecules with various nuclei the energy levels are calculated from the level of heavier isotope)

	L = 0		L = 0		L = 2	L = 3
	n = 0	n = 1	n = 0	n = 1	n = 0	n = 0
$(pp)_\mu^+$	252	–	109	–	–	–
$(dd)_\mu^+$	330	40	227	7?	88	–
$(tt)_\mu^+$	367	86	283	45	170	50
$(pd)_\mu^+$	220	–	90	–	–	–
$(pt)_\mu^+$	213	–	93	–	–	–
$(dt)_\mu^+$	318	32	232	–	102	–

* ** These values are derived for $K_{\kappa\kappa} \cong 0$. Calculations with correct value of $K_{\kappa\kappa}$ give for λ the value ~ 10, what seems to be doubtful.

T a b l e IV

probabilities of mesic molecule formation in units 10^6 sec^{-1} in liquid hydrogen

$(pp)^+_\mu$	$(dd)^+_\mu$	$(tt)^+_\mu$	$(pd)^+_\mu$	$(dt)^+_\mu$	$(pt)^+_\mu$
1.53	0.006	0.38	0.7	~0.001	0.25

In the present paper the probabilities of the mesic molecule formation schould be considered to be correct only in order of value, since the binding of hydrogen nuclei in molecules of H_2 has been neglected in our calculations what leads apparetly to the increase of W.

In the case of the mesic molecular ion (dd) and (dt) 0–0 transitions may contribute to the molecule formation probability, due to the oscillation level close to zero ($L = 0$).

We have just reseived preprints by Cohen, Judd and Riddell. They calkulate the probability of the formation of mesic molecules with identical nuclei using the wave functions normalised to $\sqrt{2}$, but not to unity, what doubles the value of W.

We wish to thank Y.B. Zeldovich for initiating this investigation and for many useful discussions.

Table V

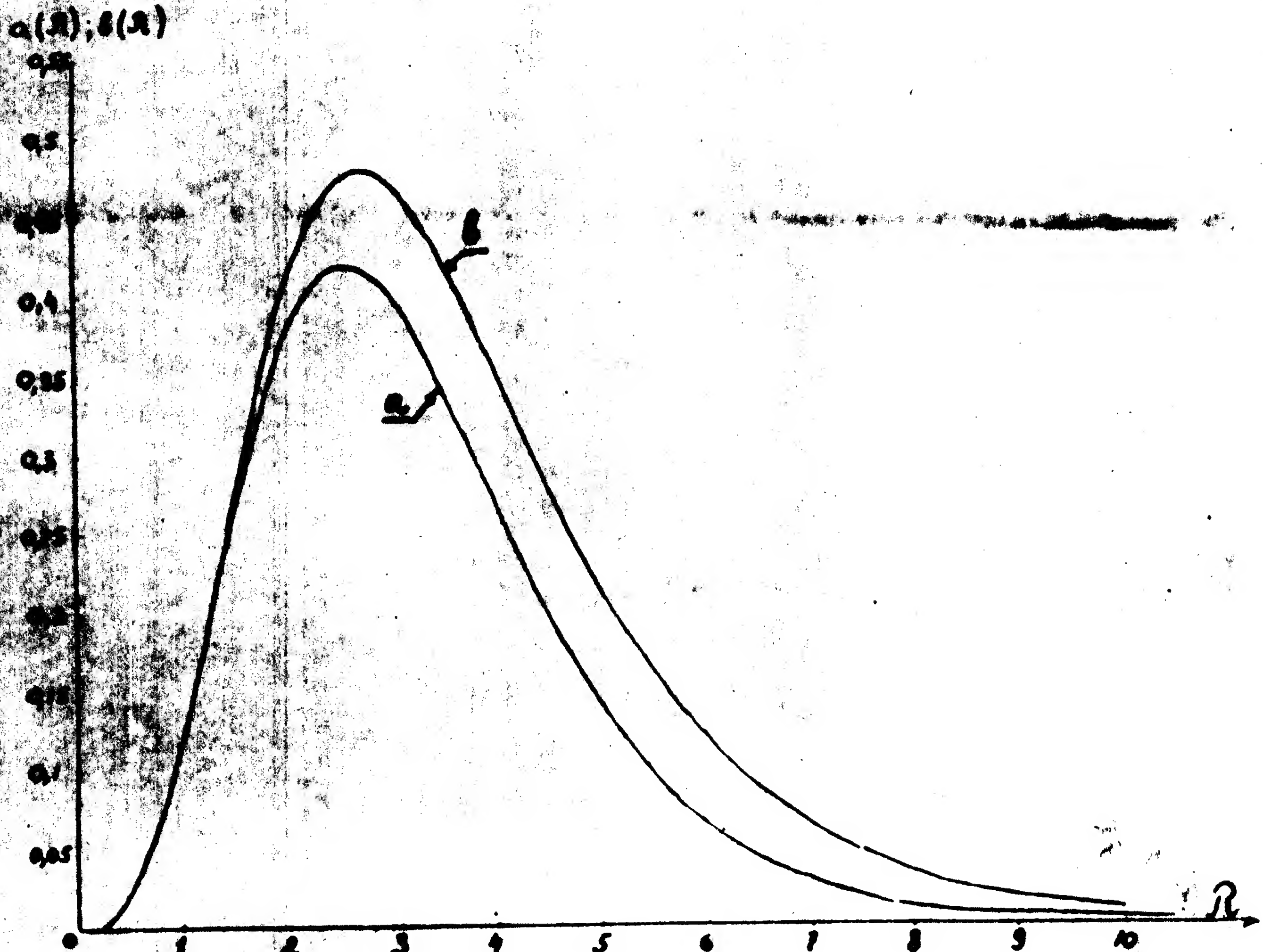
Wave functions of mesic molecular ions (unnormalized) in state L = 1

	$(pd)^{+}_{\mu}$ Level 90 ev		$(pt)^{+}_{\mu}$ Level 98 ev		$(dt)^{+}_{\mu}$ Level 23[illegible] ev		
R	a(R)	b(R)	a(R)	b(R)	a(R)	b(R)	R
0,3	0,114 10^{-2}	0,110 10^{-2}	0,116 10^{-2}	0,109 10^{-2}	+0,772 10^{-3}	+0,823 10^{-3}	0,3
0,5	0,530 10^{-2}	0,516 10^{-2}	0,554 10^{-2}	0,531 10^{-2}	0,427 10^{-2}	0,420 10^{-2}	0,5
0,7	0,132 10^{-1}	0,130 10^{-1}	0,141 10^{-1}	0,137 10^{-1}	0,122 10^{-1}	0,121 10^{-1}	0,7
0,9	0,256 10^{-1}	0,254 10^{-1}	0,277 10^{-1}	0,273 10^{-1}	0,246 10^{-1}	0,260 10^{-1}	0,9
1,1	0,423 10^{-1}	0,424 10^{-1}	0,462 10^{-1}	0,462 10^{-1}	0,464 10^{-1}	0,462 10^{-1}	1,1
1,3	0,625 10^{-1}	0,632 10^{-1}	0,687 10^{-1}	0,697 10^{-1}	0,718 10^{-1}	0,719 10^{-1}	1,3
1,5	0,849 10^{-1}	0,869 10^{-1}	0,936 10^{-1}	0,904 10^{-1}	0,100	0,101	1,5
[illegible],7	[illegible]	0,11[illegible]	0,119	0,125	0,129	0,131	1,7
1,9	0,130	0,137	0,144	0,153	0,156	[illegible]	1,9
2,1	0,151	0,160	0,165	0,179	0,179	0,183	2,1
2,3	0,168	0,181	0,183	0,202	0,196	0,202	2,3
2,5	0,182	0,198	0,197	0,221	0,207	0,214	2,5
2,7	0,191	0,211	0,206	0,235	0,213	0,219	2,7
2,9	0,197	0,221	0,210	0,245	0,210	0,219	2,9
3,1	0,198	0,226	0,210	0,250	0,203	0,213	3,1
3,3	0,196	0,228	0,205	0,252	0,193	0,204	3,3
3,5	0,191	0,226	0,198	0,249	0,179	0,191	3,5
3,7	0,184	0,221	0,189	0,243	0,164	0,176	3,7
3,9	0,175	0,215	0,177	0,234	0,148	0,160	3,9
4,1	0,164	0,206	0,164	0,224	0,131	0,143	4,1
4,3	0,152	0,196	0,151	0,212	0,115	0,127	4,3
4,5	0,140	0,184	0,137	0,200	0,100	0,111	4,5
4,7	0,128	0,173	0,124	0,187	0,866 10^{-1}	0,969 10^{-1}	4,7
4,9	0,116	0,161	0,111	0,173	0,741 10^{-1}	0,838 10^{-1}	4,9
5,1	0,104	0,149	0,989 10^{-1}	0,161	0,630 10^{-1}	0,719 10^{-1}	5,1
5,3	0,935 10^{-1}	0,137	0,875 10^{-1}	0,148	0,532 10^{-1}	0,614 10^{-1}	5,3
5,5	0,833 10^{-1}	0,126	0,770 10^{-1}	0,136	0,447 10^{-1}	0,521 10^{-1}	5,5
5,7	0,738 10^{-1}	0,115	0,673 10^{-1}	0,125	0,373 10^{-1}	0,440 10^{-1}	5,7
5,9	0,650 10^{-1}	0,105	0,586 10^{-1}	0,114	0,310 10^{-1}	0,371 10^{-1}	5,9
6,0	0,609 10^{-1}	0,100	0,546 10^{-1}	0,10[illegible]	0,283 10^{-1}	0,340 10^{-1}	6,0
6,2	0,534 10^{-1}	0,908 10^{-1}	0,471 10^{-1}	0,993 10^{-1}	0,233 10^{-1}	0,279 10^{-1}	6,2
6,4	0,465 10^{-1}	0,822 10^{-1}	0,40[illegible] 10^{-1}	0,905 10^{-1}	0,193 10^{-1}	0,237 10^{-1}	6,4
6,6	0,403 10^{-1}	0,742 10^{-1}	0,34[illegible] 10^{-1}	0,823 10^{-1}	0,158 10^{-1}	0,197 10^{-1}	6,5
6,8	0,348 10^{-1}	0,669 10^{-1}	0,290 10^{-1}	0,749 10^{-1}	0,129 10^{-1}	0,164 10^{-1}	6,8
7,0	0,299 10^{-1}	0,601 10^{-1}	0,24[illegible] 10^{-1}	0,681 10^{-1}	0,106 10^{-1}	0,135 10^{-1}	7,0
7,2	0,255 10^{-1}	0,539 10^{-1}	0,198 10^{-1}	0,618 10^{-1}	0,865 10^{-2}	0,111 10^{-1}	7,2
7,4	0,216 10^{-1}	0,483 10^{-1}	0,158 10^{-1}	0,562 10^{-1}	0,701 10^{-2}	0,910 10^{-1}	7,4
7,6	0,181 10^{-1}	0,432 10^{-1}	0,122 10^{-1}	0,511 10^{-1}	0,569 10^{-2}	0,738 10^{-2}	7,6
7,8	0,150 10^{-1}	0,385 10^{-1}	0,860 10^{-2}	0,4[illegible]4 10^{-1}	0,451 10^{-2}	0,596 10^{-2}	7,8
8,0	0,12[illegible] 10^{-1}	0,342 10^{-1}	0,557 10^{-2}	0,422 10^{-1}	0,381 10^{-2}	0,478 10^{-2}	8,0
8,5	0,677 10^{-2}	0,253 10^{-1}	0,136 10^{-2}	0,333 10^{-1}	0,312 10^{-2}	0,253 10^{-2}	8,5

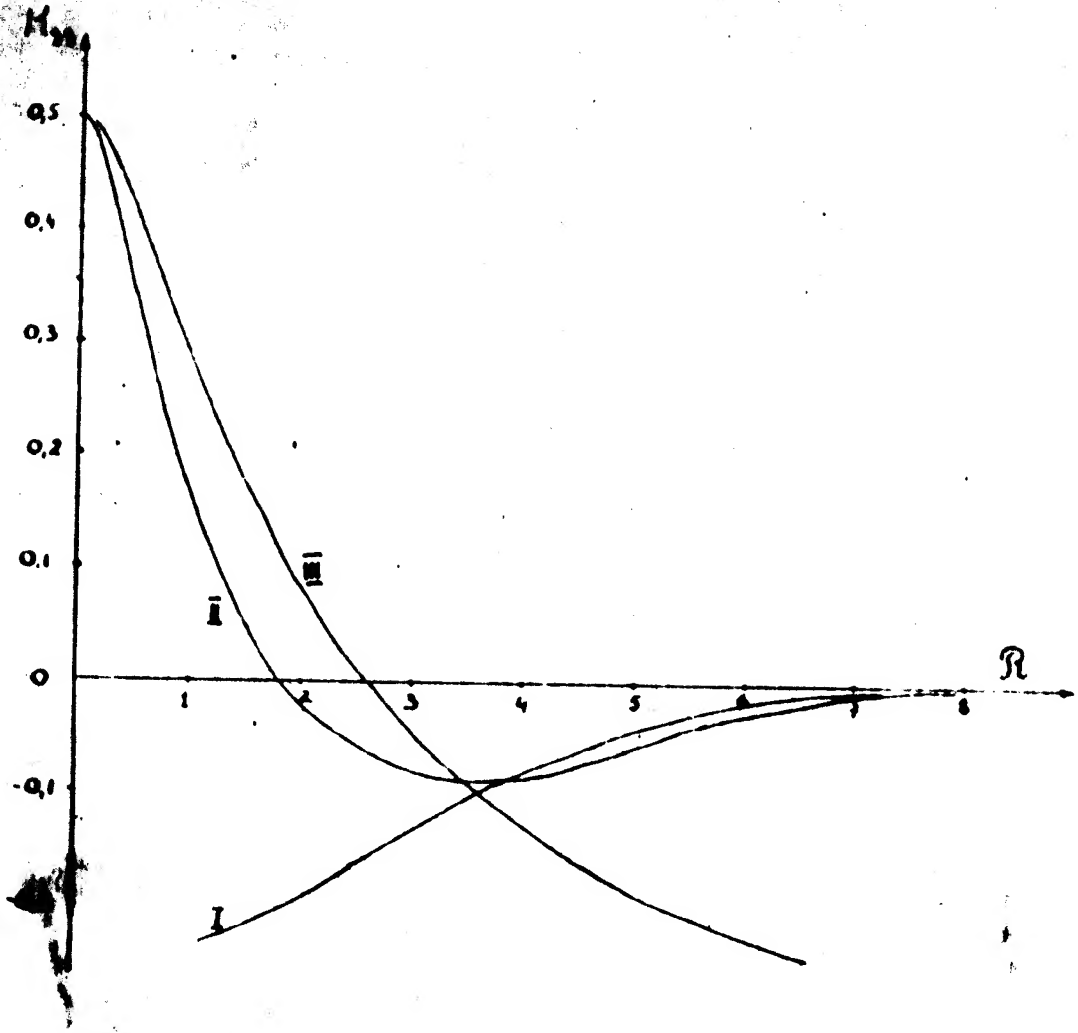
Table V (continuation)

Wave functions. Scattering of mesic atom with a heavier isotope by nuclei of lighter one at zero energy

R	$d_\mu + p$ a(R)	$d_\mu + p$ b(R)	$t_\mu + p$ a(R)	$t_\mu + p$ b(R)	$t_\mu + d$ a(R)	$t_\mu + d$ b(R)	R
0,3	-0,469 10^{-2}	-0,454 10^{-2}	-0,21[illegible] 10^{-1}	-0,164 10^{-1}	-0,[illegible]24 10^{-2}	-0,308 10^{-2}	0,3
0,5	-0,181 10^{-1}	-0,178 10^{-1}	-0,[illegible]35 10^{-1}	-0,4[illegible]3 10^{-1}	-0,362 10^{-1}	-0,377 10^{-1}	0,5
0,7	-0,386 10^{-1}	-0,382 10^{-1}	-0,950 10^{-1}	-0,935 10^{-1}	-0,946 10^{-1}	-0,939 10^{-1}	0,7
0,9	-0,655 10^{-1}	-0,655 10^{-1}	-0,1[illegible]	-0,164	-0,178	-0,178	0,9
1,1	-0,965 10^{-1}	-0,974 10^{-1}	-0,245	-0,248	-0,281	-0,281	1,1
1,3	-0,129	-0,131	-0,329	-0,337	-0,387	-0,390	1,3
1,5	-0,159	-0,163	-0,407	-0,423	-0,477	-0,487	1,5
1,7	-0,183	-0,190	-0,470	-0,4[illegible]6	-0,540	-0,550	1,7
1,9	-0,201	-0,210	-0,513	-0,549	-0,560	-0,573	1,9
2,1	-0,210	-0,221	-0,532	-0,576	-0,533	-0,55	2,1
2,3	-0,210	-0,222	-0,526	-0,577	-0,462	-0,480	2,3
2,5	-0,202	-0,213	-0,498	-0,549	-0,353	-0,371	2,5
2,7	-0,186	-0,195	-0,449	-0,496	-0,217	-0,233	2,7
2,9	-0,163	-0,168	-0,385	-0,420	-0,661 10^{-1}	-0,772 10^{-1}	2,9
3,1	-0,137	-0,135	-0,310	-0,325	+0,892 10^{-1}	+0,847 10^{-1}	3,1
3,3	-0,107	-0,948 10^{-1}	-0,229	-0,215	0,239	0,242	3,3
3,5	-0,754 10^{-1}	-0,508 10^{-1}	-0,145	-0,940 10^{-1}	0,374	0,385	3,5
3,7	-0,440 10^{-1}	-0,378 10^{-2}	-0,631 10^{-1}	+0,341 10^{-1}	0,491	0,510	3,7
3,9	-0,136 10^{-1}	+0,451 10^{-1}	+0,142 10^{-1}	0,166	0,585	0,610	3,9
4,1	+0,148 10^{-1}	0,948 10^{-1}	0,849 10^{-1}	0,299	0,655	0,685	4,1
4,3	0,407 10^{-1}	0,145	0,147	0,431	0,703	0,734	4,3
4,5	0,636 10^{-1}	0,194	0,201	0,561	0,729	0,757	5,3
4,7	0,832 10^{-1}	0,242	0,245	0,687	0,737	0,757	4,7
4,9	0,996 10^{-1}	0,289	0,280	0,808	0,729	0,737	4,9
5,1	0,113	0,334	0,306	0,925	0,709	0,698	5,1
5,3	0,123	0,378	0,325	1,04	0,679	0,644	5,3
5,5	0,130	0,420	0,337	1,15	0,642	0,577	5,5
5,7	0,135	0,461	0,342	1,25	0,600	0,499	5,7
5,9	0,138	0,500	0,343	1,35	0,556	0,413	5,9
6,0	0,139	0,519	0,341	1,40	0,533	0,367	6,0
6,2	0,139	0,556	0,335	1,49	0,487	0,270	6,2
6,4	0,137	0,593	0,326	1,58	0,443	0,170	6,4
6,6	0,134	0,628	0,314	1,67	0,399	0,653 10^{-1}	6,6
6,8	0,130	0,662	0,300	1,75	0,358	-0,421 10^{-1}	6,8
7,0	0,126	0,696	0,284	1,84	0,319	-0,151	7,0
7,2	0,121	0,729	0,269	1,92	0,284	-0,262	7,2
7,4	0,115	0,761	0,253	1,99	0,251	-0,374	7,4
7,6	0,110	0,793	0,237	2,08	0,221	-0,48[illegible]	7,6
7,8	0,104	0,824	0,221	2,15	0,194	-0,593	7,8
8,0	0,981 10^{-1}	0,855	0,205	[illegible]	0,170	-0,712	8,0
8,5	0,844 10^{-1}	0,931	0,171	[illegible]	0,1[illegible]	-0,994	8,5
9,0	0,724 10^{-1}	1,01	0,142	2,60	0,836 10^{-1}	-1,27	9,0
9,5	0,623 10^{-1}	1,08	0,116	2,78	0,571 10^{-1}	-1,55	9,5
10,0	0,545 10^{-1}	1,15	0,939 10^{-1}	2,95	0,385 10^{-1}	-1,83	10,0
10,5	0,488 10^{-1}	1,22	0,739 10^{-1}	3,13	0,259 10^{-1}	-2,11	10,5
11,0	0,457 10^{-1}	1,30	0,538 10^{-1}	3,31	0,176 10^{-1}	-2,38	11,0
11,5	0,453 10^{-1}	1,37			0,122 10^{-1}	-2,66	11,5
12,0	0,476 10^{-1}	1,44			0,834 10^{-2}	-2,94	12,0
					0,[illegible]8 10^{-2}	-3,49	13



F i g. 1. Wave functions of mesic molecular ion $(pt)_{\mu}$ corresponding to the energy level 96 ev. The functions are given for illustration of qualitative behaviour of a (R) and b (R) for discrete values E. (Exact values of function are given in table V).



F i g. 2. *Function K_{gg} calculated: I-in (LCAO)-approximation; II - with exact functions; III - in (UA) -approximation.*

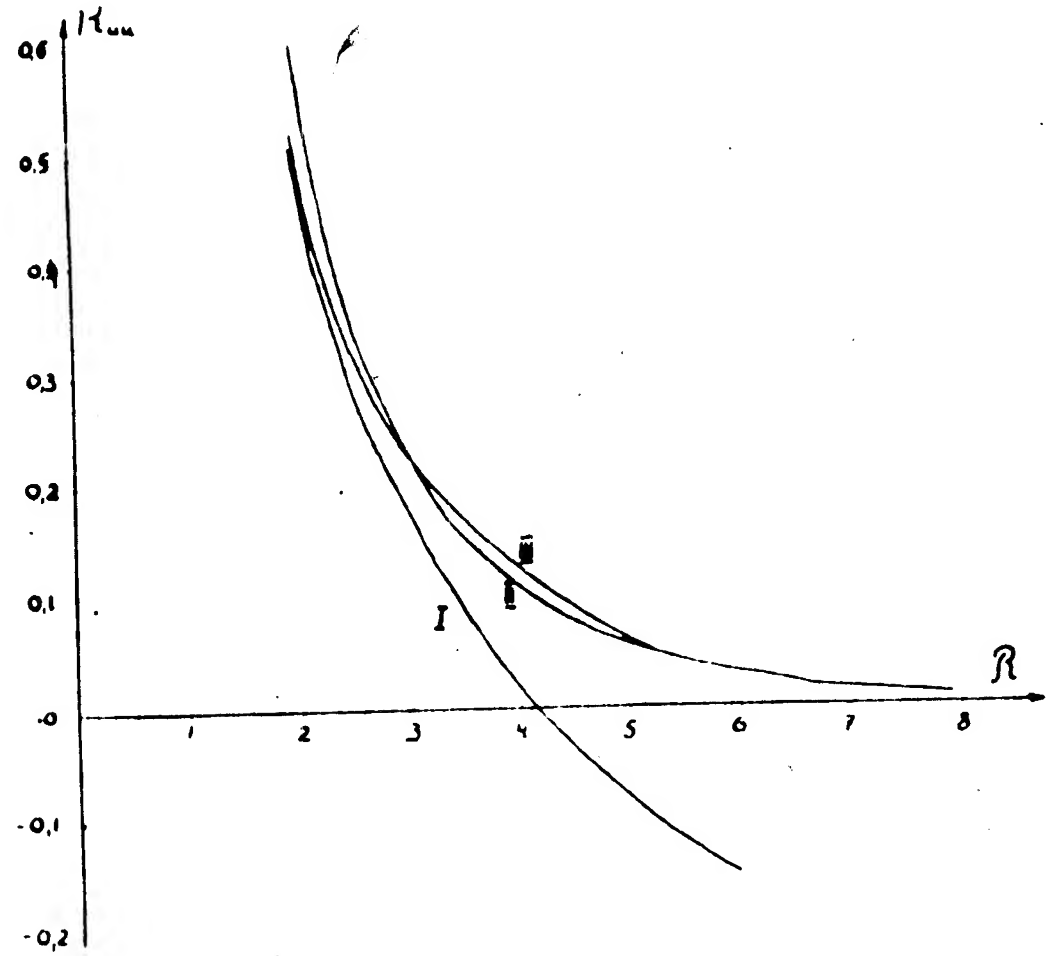


Fig. 3. *Function Kuu calculated: I - in (UA)-approximation; II - in (LCAO) approximation; III - with exact functions* /15/.

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